A comparison of search templates for gravitational waves from binary inspiral — 3.5PN update

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Phasing formulas in [1] are updated taking into account the recent 3.5PN results. Some misprints in the published version of [1] are also corrected.

I. INTRODUCTION

In this note we update Tables I and II in Ref. [1] (henceforth referred to as DIS3) in view of the recent theoretical progress made in the dynamics of, and radiation from, binary systems to 3.5PN order [2, 3, 4, 5]. Before giving a comprehensive list of the corresponding updates, we take this opportunity to correct some misprints in Ref. [1]. Results in [1] are not changed since they used the correct formulas free of the misprints below.

The expansion coefficients in Table I and Table II of DIS3, are all Newton-normalised coefficients. In the notation of the paper, there should be overhats on all these coefficients, except e_k (that is, $\widehat{E}_k$, $\widehat{\mathcal{F}}_k$, $\hat{t}^v_k$, $\hat{\phi}^v_k$, $\hat{\phi}^t_k$, $\widehat{F}^t_k$, and $\hat{\tau}_k$). The coefficients $\hat{t}^v_5$, $\widehat{F}^t_5$ and $\hat{\tau}_2$ in Table II (which were correct in the eprint version) contain typographical errors in the published version of DIS3. Their correct expressions are:

$$\hat{t}^v_5 = -\left(\frac{7729}{252} - \frac{13}{3}\eta\right)\pi, \tag{1.1}$$

$$\widehat{F}^t_5 = -\left(\frac{7729}{21504} - \frac{13}{256}\eta\right)\pi, \tag{1.2}$$

$$\hat{\tau}_2 = \frac{5}{9}\left(\frac{743}{84} + 11\eta\right). \tag{1.3}$$

The second of the equations in Eq. (4.5) of DIS3 should read

$$p^0_\varphi = \left[\frac{r_0^2 - 3\eta}{r_0^3 - 3r_0^2 + 5\eta}\right]^{1/2} r_0, \tag{1.4}$$

the factor r_0 outside the square brackets on the right-hand side is missing both in the eprint and published version of DIS3.

II. UPDATES

The energy [2, 3, 4] and flux [5] functions have now been computed up to order v^7 in post-Newtonian theory. The corresponding expansion coefficients are as follows. The 3PN coefficients in the expansion of the various energy functions are:

$$\begin{aligned}\widehat{E}_3 &= -\frac{675}{64} + \left[\frac{34445}{576} - \frac{205\pi^2}{96} + \frac{10\omega_s}{3}\right]\eta \\ &- \frac{155}{96}\eta^2 - \frac{35}{5184}\eta^3,\end{aligned} \tag{2.1}$$

$$\begin{aligned}e_3 &= -9 + \left(\frac{4309}{72} - \frac{205}{96}\pi^2 + \frac{10}{3}\omega_s\right)\eta \\ &- \frac{103}{36}\eta^2 + \frac{1}{81}\eta^3,\end{aligned} \tag{2.2}$$

with the Padé approximant e_{P_6} determined using Eq. (2.17) of DIS3, wherein c_1 and c_2 are as in DIS3 and c_3 is given by

$$c_3 = \frac{e_1 e_3 - e_2^2}{e_1\left(e_1^2 - e_2\right)}. \tag{2.3}$$

The dimensionless parameter ω_s (used in [2]) is related to the parameter λ (used in [3]) by $\omega_s = -1987/840 - 11\lambda/3$, so that we alternatively have

$$\begin{aligned}\widehat{E}_3 &= -\frac{675}{64} + \left[\frac{209323}{4032} - \frac{205\pi^2}{96} - \frac{110\lambda}{9}\right]\eta \\ &- \frac{155}{96}\eta^2 - \frac{35}{5184}\eta^3,\end{aligned} \tag{2.4}$$

$$\begin{aligned}e_3 &= -9 + \left(\frac{26189}{504} - \frac{205}{96}\pi^2 - \frac{110}{9}\lambda\right)\eta \\ &- \frac{103}{36}\eta^2 + \frac{1}{81}\eta^3.\end{aligned} \tag{2.5}$$

The numerical value of ω_s has been recently determined by dimensional regularization [4] to be simply equal to

$\omega_s = 0$, which corresponds to $\lambda = -1987/3080$. [Note that there is a sign misprint in the second term on the right-hand-side of Eq.(4.7) in the last reference in [2]; it should read $\lambda = -3\omega_s/11 - 1987/3080$.] Concerning the 3PN update of the effective one-body Hamiltonian, it is explicitly given in section IVD of the second reference in [2].

The expansion coefficients in Table II of DIS3 at 3PN and 3.5PN are as follows. The coefficients in the expansion of the flux are:

$$\begin{aligned}\widehat{\mathcal{F}}^v_6 &= \frac{6643739519}{69854400} + \frac{16\pi^2}{3} - \frac{1712}{105}\gamma \\ &+ \left(-\frac{11497453}{272160} + \frac{41\pi^2}{48} + \frac{176}{9}\lambda - \frac{88}{3}\Theta\right)\eta \\ &- \frac{94403}{3024}\eta^2 - \frac{775}{324}\eta^3, \end{aligned} \tag{2.6}$$

$$\widehat{\mathcal{F}}^v_7 = \left(-\frac{16285}{504} + \frac{214745}{1728}\eta + \frac{193385}{3024}\eta^2\right)\pi, \tag{2.7}$$

and $\widehat{\mathcal{F}}^v_{l6} = -\frac{1712}{105}$. Here γ is the Euler constant, $\gamma = 0.577\ldots$, and Θ and λ are two undetermined parameters in Ref. [5]. We use the letter Θ to denote what is denoted by θ in [5] (this should not be confused with the related undetermined 3PN quantity $\hat{\theta} = \theta - 7\lambda/3$ which is also used in some formulas of [5]). The λ appearing in the flux function is the same quantity as in the energy function, arising, as it does, from the time derivatives of the mass quadrupole moment involved in computing the far-zone flux. The quantity $\widehat{F}^v_{l6}$ is the coefficient of the log term that arises, for the first time, at the 3PN order; to the usual *Newton-normalized* Taylor expansion [6] one must add $\widehat{\mathcal{F}}^v_{l6}\log(4v)v^6$ to complete the PN expansion. Beware that if expressions are rewritten in terms of ω_s rather than λ, the rational numerical coefficient in the η term will change. For ready reckoning, in the above and subsequent formulas, we indicate this explicitly as follows: $(-11497453/272160 + (176\lambda)/9 \to -14930989/272160 - (16\omega_s)/3)$. This means the flux formula may be alternatively written in terms of ω_s by the indicated replacement.

Coefficients in the expansion of time as a function of the invariant velocity parameter $v = (\pi m f)^{1/3}$, where f is the gravitational-wave frequency, are

$$\begin{aligned}\hat{t}^v_6 &= -\frac{10052469856691}{23471078400} + \frac{128}{3}\pi^2 \\ &+ \left(\frac{15335597827}{15240960} - \frac{451}{12}\pi^2 + \frac{352}{3}\Theta - \frac{2464}{9}\lambda\right)\eta \\ &+ \frac{6848}{105}\gamma - \frac{15211}{1728}\eta^2 + \frac{25565}{1296}\eta^3, \end{aligned} \tag{2.8}$$

$$\hat{t}^v_7 = \left(-\frac{15419335}{127008} - \frac{75703}{756}\eta + \frac{14809}{378}\eta^2\right)\pi, \tag{2.9}$$

and $\hat{t}^v_{l6} = \frac{6848}{105}$, where $\hat{t}^v_{l6}$ is the coefficient of the log term; to the usual Newton-normalized Taylor expansion one must add $\hat{t}^v_{l6}\log(4v)v^6$ to complete the PN expansion. $(15335597827/15240960 - (2464\lambda)/9 \to 18027490051/15240960 + (224\omega_s)/3)$.

Coefficients in the expansion of the gravitational wave phase as a function of the invariant velocity v are

$$\begin{aligned}\hat{\phi}^v_6 &= \frac{12348611926451}{18776862720} - \frac{160}{3}\pi^2 - \frac{1712}{21}\gamma \\ &+ \left(-\frac{15335597827}{12192768} + \frac{2255}{48}\pi^2 + \frac{3080}{9}\lambda - \frac{440}{3}\Theta\right)\eta \\ &+ \frac{76055}{6912}\eta^2 - \frac{127825}{5184}\eta^2, \end{aligned} \tag{2.10}$$

$$\hat{\phi}^v_7 = \left(\frac{77096675}{2032128} + \frac{378515}{12096}\eta - \frac{74045}{6048}\eta^2\right)\pi, \tag{2.11}$$

and $\hat{\phi}^v_{l6} = -\frac{1712}{21}$, where, as in the previous cases, $\hat{\phi}^v_{l6}$ is the coefficient of the log term; to the usual Newton-normalized Taylor expansion one must add $\hat{\phi}^v_{l6}\log(4v)v^6$ to complete the PN expansion. $(-15335597827/12192768 + (3080\lambda)/9 \to -18027490051/12192768 - (280\omega_s)/3)$.

The expansion coefficients of the phase as a function of the time parameter[7] $\theta = [\eta(t_{\rm ref} - t)/(5m)]^{-1/8}$ where $t_{\rm ref}$ is a reference time at which the PN-expanded GW frequency formally goes to infinity, are given by,

$$\begin{aligned}\hat{\phi}^t_6 &= \frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}\gamma \\ &+ \left(-\frac{123292747421}{4161798144} + \frac{2255}{2048}\pi^2 + \frac{385}{48}\lambda - \frac{55}{16}\Theta\right)\eta \\ &+ \frac{154565}{1835008}\eta^2 - \frac{1179625}{1769472}\eta^3. \end{aligned} \tag{2.12}$$

$$\hat{\phi}^t_7 = \left(\frac{188516689}{173408256} + \frac{488825}{516096}\eta - \frac{141769}{516096}\eta^2\right)\pi, \tag{2.13}$$

and $\hat{\phi}^t_{l6} = -\frac{107}{56}$, where, as before, $\hat{\phi}^t_{l6}$ is the coefficient of the log term; to the usual Newton-normalized Taylor expansion one must add $\hat{\phi}^t_{l6}\log(2\theta)\theta^6$ to complete the PN expansion. $(-123292747421/4161798144 + (385\lambda)/48 \to -144827885213/4161798144 - (35\omega_s)/16)$.

Finally, coefficients in the expansion of the gravitational-wave frequency in terms of the time parameter θ are given by

$$\begin{aligned}\widehat{F}^t_6 &= -\frac{720817631400877}{288412611379200} + \frac{53}{200}\pi^2 + \frac{107}{280}\gamma \\ &+ \left(\frac{123292747421}{20808990720} - \frac{451}{2048}\pi^2 - \frac{77}{48}\lambda + \frac{11}{16}\Theta\right)\eta \\ &- \frac{30913}{1835008}\eta^2 + \frac{235925}{1769472}\eta^3, \end{aligned} \tag{2.14}$$

$$\widehat{F}^t_7 = \left(-\frac{188516689}{433520640} - \frac{97765}{258048}\eta + \frac{141769}{1290240}\eta^2\right)\pi. \tag{2.15}$$

and $\widehat{F}^t_{l6} = \frac{107}{280}$, where, $\widehat{F}^t_{l6}$ is the coefficient of the log term; to the usual Newton-normalized Taylor expansion one must add $\widehat{F}^t_{l6} \log(2\theta)\theta^6$ to complete the PN expansion. $(123292747421/20808990720 - (77\lambda)/48 \rightarrow 144827885213/20808990720 + (7\omega_s)/16)$.

In computing Pade coefficients of the *new* flux function [6] one needs the first seven continued fraction coefficients. The first six of these are as in Appendix A of Ref. [6], except that the last term in the first line of c_6 should be $c_3^2(c_2 - c_1)$. The coefficient c_7 is too long to be quoted in this brief note; an electronic version can be obtained from the authors.

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[7] The dimensionless time variable θ here is related to τ in [3], by $\theta = \tau^{-1/8}$ leading to minor differences in the coefficients here and in [3].